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NUCLEAR ELASTICITY APPROACH TO GIANT RESONANCES

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Résumé - La fragmentation des résonances géantes monopolaire et quadrupolaire isoscalaires dans les noyaux déformés a été étudiée dans le cadre de l'élasticité nucléaire.

Abstract - The fragmentation of the isoscalar giant monopole and quadrupole resonances in deformed nuclei has been studied within the framework of the nuclear elasticity.

The present communication is concerned with the isoscalar giant resonances in deformed nuclei, in particular the fragmentation of the giant monopole and quadrupole resonances. The model in which we have worked is the nuclear elasticity which has been proposed initially by G. Bertsch /1/ and also by C.Y. Wong /2/. The conception of the nuclear elasticity is still unfamiliar with nuclear physicists but I will not go into the discussion on the foundation of this model in the present contribution.

Because the monopole operator has no directional projection, the giant monopole resonance can not be split into fragments in deformed nuclei. However, it has been observed /3/ that the giant monopole resonance has two components in the ^{154}Sm nucleus. The coupling between the monopole and quadrupole oscillations by means of nuclear deformation parameters can provide an explanation of this phenomenon.

The nuclear elasticity approach to nuclear collective motions, such as the giant resonances, is essentially to consider the nuclear matter as an elastic solid body which can vibrate as a whole. As a matter of fact, the fundamental equation of motion in this approach is the socalled Lamé equation for uniform, perfectly elastic medium:

$$(\lambda+2\mu)\vec{\nabla}(\vec{\nabla}\cdot\vec{u}) - \mu\vec{\nabla}\times\vec{\nabla}\times\vec{u} + \vec{F} = \rho\,\frac{\partial^2\vec{u}}{\partial t^2}$$

where λ and μ the Lamé coefficients, $\vec{u}$ the displacement vector in the elastic medium, $\vec{F}$ the body force and ρ the density. For the nuclear elastic medium the Lamé coefficients are shown to be

$$\mu = \frac{\hbar^2}{5m}k_f^2\rho\ , \qquad \lambda = \left(\frac{K}{9} - \frac{2}{15}\frac{\hbar^2}{m}k_f^2\right)\rho\ ,$$

where m is the effective nucleon mass, k_f the Fermi momentum, and K the nuclear compressibility. The problem is now to solve the Lamé equation under appropriate boundary conditions. For exemple, we can impose a simple boundary condition which states that the stress components vanish on the nuclear surface. This boundary condition together with the assumption of constant density yields an eigenvalue equation for free vibrations of spherical nuclei. This eigenvalue equation takes the form

$$2\,\frac{\xi}{\eta}\left[\frac{1}{\eta} + \frac{(\ell-1)(\ell+2)}{\eta^2}\left\{\frac{j_{\ell+1}(\eta)}{j_\ell(\eta)} - \frac{\ell+1}{\eta}\right\}\right]j_{\ell+1}(\xi) +$$

$$+ \left[-\frac{1}{2} + \frac{(\ell-1)(\ell+2)}{\eta^2} + \frac{1}{\eta}\left\{ 1 - \frac{2\ell(\ell-1)(\ell+2)}{\eta^2} \right\} \frac{j_{\ell+1}(\eta)}{j_\ell(\eta)} \right] j_\ell(\xi) = 0 \quad ,$$

where $\xi^2 = \frac{\rho\omega^2}{\lambda + 2\mu} R_o^2$ and $\eta^2 = \frac{\rho\omega^2}{\mu} R_o^2$, ω being the frequency of vibration.

This eigenvalue equation may be compared with a corresponding equation which arises from the well known boundary condition in the hydrodynamical model, namely, no flows across the nuclear surface R_o . This statement leads to

$$\frac{\ell}{kR_o} j_\ell(kR_o) - j_{\ell+1}(kR_o) = 0,$$

where k is the wave number in the hydrodynamical equation. Once the eigenvalues are known, the giant resonance energies for spherical nuclei can be evaluated using the numerical values of the Lamé coefficients for which use of $K=K_\infty=220$ MeV and $k_f = 1.3\text{fm}^{-1}$ yield a correct order of magnitude. For a more precise calculation, it is necessary to introduce a realistic parametrization of these quantities as well as the effective nucleon mass. Modification of the boundary condition so as to include surface tension and Coulomb interaction improves certainly the numerical results of the giant resonance energies.

For deformed nuclei, however, the problem is more complicated owing to the difficulty of solving the Lamé equation in spheroidal coordinate systems. One method of overcoming this difficulty is to apply first the variational principle to the equation of motion and then solve it by assuming a small deviation of nuclear figure from that of the spherical nucleus. For exemple, the variational expression for the scalar Helmholtz equation

$$\vec{\nabla}\psi^2 + k^2\psi = 0,$$

is simply

$$k^2 \leqslant \frac{\int |\vec{\nabla}\psi|^2 d\tau}{\int |\psi|^2 d\tau} \quad ,$$

where k is the wave number. However, the procedure of writing down a similar variational expression from the Lamé equation is not straightforward. After a rather lengthy calculation, we arrive at the expression

$$\omega^2 \leqslant \frac{\lambda \int |\operatorname{div} \vec{u}|^2 d\tau + 2\mu \sum_{ij}^{3} \int |e_{ij}|^2 d\tau}{\int \rho \, |\vec{u}|^2 d\tau} \quad ,$$

where e_{ij} are the strain tensors expressed here in the spherical polar coordinates. For exemple,

$$e_{12} = e_{21} = e_{r\theta} = \frac{1}{2} \left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{1}{r} u_\theta \right) ,$$

where u_r and u_θ are respectively the r and θ components of the displacement vector $\vec{u}$.

For constant density, the Lamé equation has an analytical solution. When we introduce this solution as trial function into the variational equation, then the right hand side reduces back to the initial frequency for spherical nuclei. For deformed nuclei, the upper limit of radial integrals involved in the variational equation is not a constant radius but a deformed one which is a function of angles as well as deformation parameters. Therefore, the explilcit evaluation of the right hand side gives an expression which contains the nuclear deformation parameters in addition to the initial oscillation frequency. The nuclear radius is now $R = R_o(1 + \varepsilon_i)$, where ε_i are the increments. Assuming a quadrupole deformation, R can be expressed in terms of collective variables $\alpha_{2\nu}$ as

$$\varepsilon_{1(2)} = -\sqrt{\frac{5}{16\pi}} (\alpha_{20} \mp \sqrt{6}\alpha_{22}) \quad , \quad \varepsilon_3 = \sqrt{\frac{5}{4\pi}} \alpha_{20} \quad .$$

When we introduce the deformation parameters β and γ, the ε_i becomes

$$\varepsilon_i = \sqrt{\frac{5}{4\pi}}\,\beta\cos(\gamma - \frac{2}{3}i\pi\ ).$$

Having performed all angular integrals, we get a simple result of ω^2 for deformed nuclei;

$$\omega^2 = \frac{c_\ell + d_\ell \zeta_{\ell m}}{a_\ell + b_\ell \zeta_{\ell m}} \quad ,$$

where a_ℓ, b_ℓ, c_ℓ and d_ℓ are the resulting radial integrals evaluated up to first order of the collective variables $\alpha_{2\nu}$. The $\zeta_{\ell m}$ is the geometrical factor arising from the nuclear deformation and which takes the form

$$\zeta_{\ell m} = \frac{1}{(2\ell-1)(2\ell+3)}[\{\ell(\ell+1)-3m^2\}\varepsilon_3 + \frac{1}{2}\ell(\ell+1)\{\varepsilon_3 + 2\varepsilon_{1(2)}\}\delta_{m1}] \quad .$$

The geometrical factor vanishes for the monopole, that is for ℓ and m equal to zero. Therefore, the statical deformation alone has no effect on the giant monopole resonance, as was expected.

Fig.1 shows an exemple of the splitting of the isoscalar giant quadrupole resonance in axially deformed nuclei. Here the ordinate indicates the ratio of the oscillation frequency of deformed nuclei, ω_2', to that of spherical nuclei, ω_2. As we see, the giant quadrupole resonance is split into three components according to the values of m. It is noted that in the polar diagram for the deformation parameters β and γ, the points lying on the axes correspond to axially symmetric shapes and the six different points, one in each sector, which are obtained by reflection in the axes, represent the same shape of the nucleus. Therefore, the sector ($\gamma=0, \gamma=\pi/3$) is equivalent to the sector ($\gamma=-2\pi/3, \gamma=\pi$) in the polar diagram. Use of the values $\gamma=0$ and π simplifies the numerical calculation.

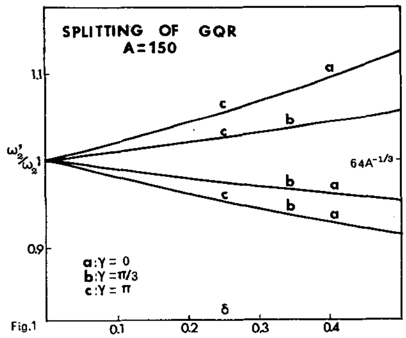

Fig.1

The coupling of the monopole oscillation with that of the quadrupole can now be achieved by introducing a trial function of the type

$$\vec{u} = \Gamma_{00}\,\vec{u}_{00} + \Gamma_{20}\,\vec{u}_{20} + \Gamma_{22}\,\vec{u}_{22}\ ,$$

where $\vec{u}_{\ell m}$ are the solutions of the Lamé equation prior to coupling and $\Gamma_{\ell m}$ are the variational parameters. The first term describes the monopole oscillation and two other terms represent the quadrupole oscillations. When we introduce this trial function into the variational equation we get

$$(\sum_{\ell m,\ell' m'} \Gamma_{\ell m}\Gamma_{\ell' m'}\cdot f_{\ell m,\ell' m'})\omega^2 - (\sum_{\ell m,\ell' m'} \Gamma_{\ell m}\Gamma_{\ell' m'}\cdot g_{\ell m,\ell' m'})\omega_0^2 = 0,$$

where $f_{\ell m,\ell' m'}$ and $g_{\ell m,\ell' m'}$ are the results of integrals evaluated also up to first order of collective variables. The frequency ω arises from the coupling and ω_0 is the initial frequency before coupling. We now remark that the differentiation of this equation with respect to the variational parameters $\Gamma_{\ell m}$ must vanish in accordance with the requirement of the variational principle. As a consequence, we obtain three linear, homogeneous equations for $\Gamma_{\ell m}$;

$$a_{11}\Gamma_{00} + a_{12}\delta\cos\gamma\Gamma_{20} + a_{13}\delta\,\sin\gamma\Gamma_{22} = 0,$$

$$a_{21}\delta\,\cos\gamma\Gamma_{00} + (a_{22}+b_{22}\delta\,\cos\gamma)\Gamma_{20} + a_{23}\delta\,\sin\gamma\Gamma_{22} = 0,$$

$$a_{31}\delta \sin\gamma\Gamma_{00} + a_{32}\delta \sin\gamma\Gamma_{20} + (a_{33}+b_{33}\delta \cos\gamma)\Gamma_{22} = 0,$$

where a_{ij} and b_{ij} are various factors resulting from factorizations of terms after differentiation with respect to $\Gamma_{\ell m}$ and contain the square of ω. Here we have used the Nilsson deformation parameter δ in stead of β . This system of equations has a non-zero solution if and only if the determinant constructed with the factors before the variational parameters, Γ_{00}, Γ_{20} and Γ_{22} vanishes. Therefore, the determinantal equation leads to a third order equation of ω^2;

$$A(\omega^2)^3 + B(\omega^2)^2 + C(\omega^2) + D = 0.$$

This cubic equation gives generally three real solutions for physically meaningful values of the deformation parameters δ and γ.

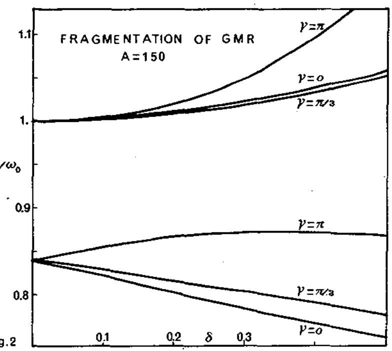

Fig.2

Fig.2 shows the result of the coupling for three values of γ, namely 0° , 60° and 180°. The numerical calculation has been performed using the modified eigenvalue equation which includes both surface tension and Coulomb energy, and the parameters for the Lamé coefficients are those of ref2. We see immediatly that the giant monopole resonance in deformed nuclei has now two components, higher and lower. The higher component is almost at the same energy as the initial energy before coupling, whereas the lower component is near $64A^{-1/3}$ MeV which is the giant quadrupole resonance energy for spherical nuclei. It is to be remarked that the coupling is meaningful for sufficiently large values of δ and this is seen from the fact that the coupling strength depends mainly on the deformation parameter δ. Fig.3 shows the same calculation but for the region of A=230. The general feature is very similar to that of the region of A=150. Fig. 4 displays the results of coupling in energy units for a fixed value of δ, 0.3. The giant monopole resonance energy before coupling is about at $80A^{-1/3}$ MeV. After coupling, the lower component is now at $63A^{-1/3}$ MeV for prolate nuclei, whereas the higher component is at $82A^{-1/3}$ MeV which is not very much different from the initial value before coupling. The positions of energies for γ=60° and 180° are also shown.

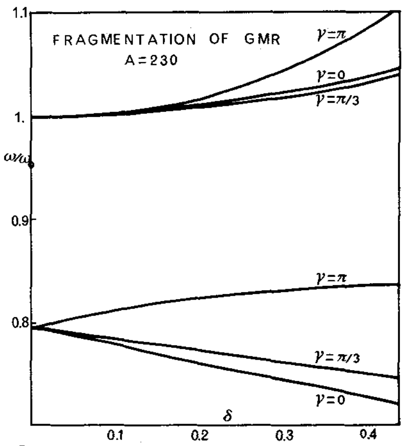

Fig.3

The coupling between the monopole and quadrupole oscillations affects also the giant quadrupole resonance. Fig.5 shows how the giant quadrupole resonance energies,which are already split before coupling, change their positions after coupling. Contrary to the giant monopole resonance, we have now a higher component which is new. Apart from the higher component, the other components have no substantial difference from the iniltial positions.

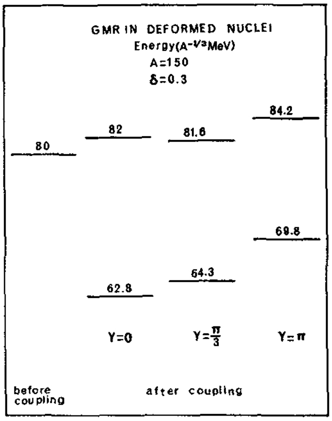

Fig.4

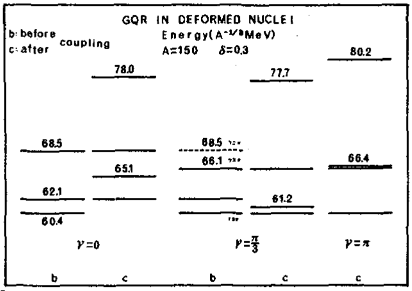

Fig.5

A detailed description of the nuclear elasticity approach to giant resonances of deformed nuclei, including both statical deformation and fast nuclear rotation, will be published elsewhere.

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